

Chiral Vortical Effect and Wigner Function

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Chiral phenomena in relativistic hydrodynamics

Investigation of Chiral Magnetic Effect (CME) and Chiral Vortical Effect (CVE)

- *A. Vilenkin, Phys. Rev. D 21 (1980) 2260; Phys. Rev. D 22 (1980) 3080.*
- *K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys.Rev. D 78, 074033 (2008).*
- *D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601.*
- *A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025.*
- *K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).*
- *M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).*
- *O. Rogachevsky, A. Sorin and O. Teryaev, Phys. Rev. C 82 (2010) 054910.*
- *Karpenko, I. et al. Eur.Phys.J. C77 (2017) no.4, 213.*
- *F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, no. 1, 012302 (2018).*

Some of the early works

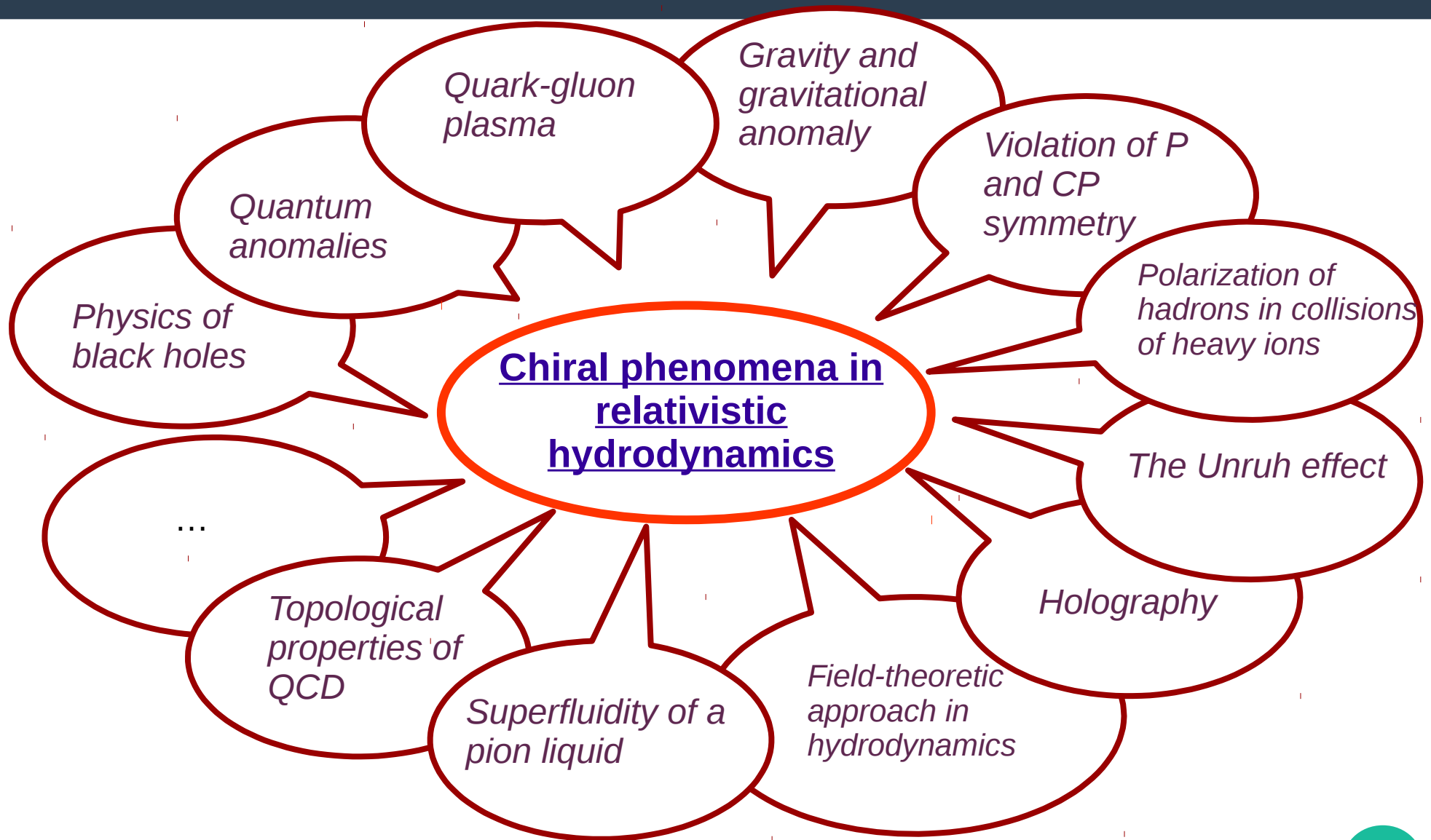
Connection with quantum anomalies

Polarisation in HIC

Some applications of the theory of chiral liquids

- Hadron polarisation in heavy ion collisions.
- Chiral batteries.

Areas of physics manifested in chiral phenomena



Achievement of D. T. Son and P. Surowka

D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601. (formulas on the slide)

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York (1959).

The law of conservation of energy/momentum in an external electromagnetic field

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

+

Quantum anomaly in divergence axial current

$$\partial_\mu j^\mu = C E^\mu B_\mu$$

Second law of thermodynamics $\partial_\mu s^\mu \geq 0$



Ideal fluid

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu},$$

$$j^\mu = n u^\mu + \nu^\mu,$$

Viscosity + electrical conductivity
+ axial anomaly

Chiral vortical effect CVE

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

$$s^\mu = s u^\mu - \frac{\mu}{T} \nu^\mu + D \omega^\mu + D_B B^\mu,$$

Chiral magnetic effect CME

$$\xi = C \left(\mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right)$$

Chiral phenomena as manifestations of an axial anomaly in effective field theory

A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, *Phys. Rev. D* 83 (2011) 105025. (formulas on the slide)

Chemical potential as an effective external electromagnetic field $qA^\mu \rightarrow \mu u^\mu$



was considered in many papers

Hydrodynamics as an effective field theory

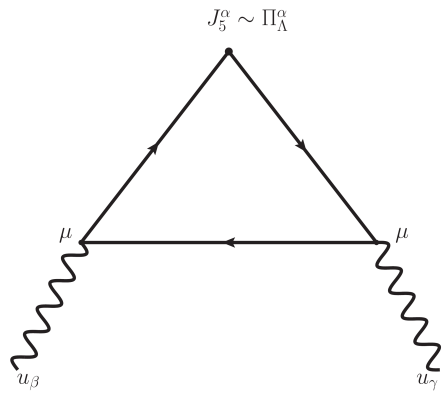
$$S_{eff} = \int dx \left(i\bar{\psi}\gamma^\rho D_\rho\psi + \mu u_\mu \bar{\psi}\gamma^\mu\psi + \mu_5 u_\mu \bar{\psi}\gamma^\mu\gamma_5\psi \right)$$



Axial anomaly in hydrodynamics

$$\partial_\mu j_5^\mu = -\frac{1}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} \left(\partial^\mu (A^\nu + \mu u^\nu) \partial^\alpha (A^\beta + \mu u^\beta) + \partial^\mu \mu_5 u^\nu \partial^\alpha \mu_5 u^\beta \right)$$

$$\partial_\mu j^\mu = -\frac{1}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\mu (A^\nu + \mu u^\nu) \partial^\alpha \mu_5 u^\beta$$



A. Sorin and O. Teryaev, *Phys. Rev. C* 95, no. 1, 011902 (2017)

Chiral effects in vector current

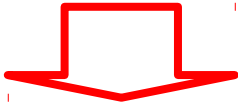
$$j^\mu = n u^\mu + \frac{\mu\mu_5}{\pi^2} \omega^\mu + \frac{\mu_5}{2\pi^2} B^\mu$$

Chiral effects in axial current

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2} \omega^\mu + \frac{\mu}{2\pi^2} B^\mu$$

Accounting for gravitational anomaly

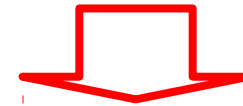
- *K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).*
- *M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).*
- *S. P. Robinson, F. Wilczek Phys.Rev.Lett. 95 (2005) 011303 MIT-CTP-3561 gr-qc/0502074.*



While the gravitational chiral anomaly is negligible in the bulk, it turns to be crucial on the edge, or on the horizon

Chiral effects in axial current

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2} \omega^\mu + \frac{\mu}{2\pi^2} B^\mu$$



Chiral vortical effect (CVE)

$$j_5^\lambda = \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2} \right) \omega^\lambda$$

Literature

- **F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, *Annals Phys.* 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].**
- **M. Buzzegoli, E. Grossi and F. Becattini, *JHEP* 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].**
- **G. Prokhorov and O. Teryaev, *Phys. Rev. D* 97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].**
- **G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to *Phys. Rev. D*.**
- **G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, arXiv:1807.03584 [hep-th].**

Problems and methods

Problems

- *Investigation of the effects of rotation and acceleration in the axial current.*
- The manifestations of the Unruh effect in the axial current and boundary temperature.
- Polarization of Λ -hyperons in collisions of heavy ions.
- Effects of finite mass.

Methods

- Covariant Wigner function for systems characterized by a thermal vorticity tensor.
- Summation of the complete series in the expansion of the mean value of the operator with respect to thermal vorticity - **nonperturbative result.**

Thermal vorticity tensor

$$\beta_\mu = \frac{u_\mu}{T}$$

4-vector of inverse temperature

Thermal vorticity tensor

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Global thermodynamic equilibrium with nonstationary movement

$$b_\mu = \text{const} \quad \varpi_{\mu\nu} = \text{const} \quad \beta_\mu = b_\mu + \varpi_{\mu\nu}x_\nu$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

(thermal) Acceleration

$$\alpha_\mu = \varpi_{\mu\nu}u^\nu$$

$$\alpha_\mu = \frac{a_\mu}{T}$$

(thermal) Vorticity

$$w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu\varpi^{\alpha\beta}$$

$$w_\mu = \frac{\omega_\mu}{T}$$

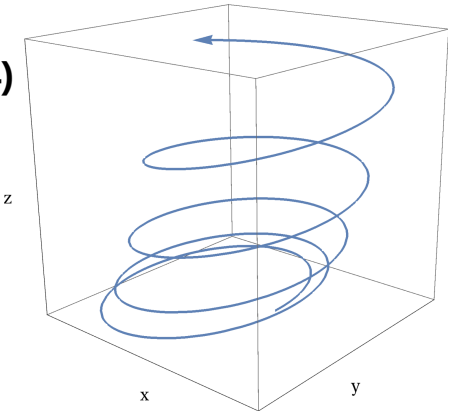
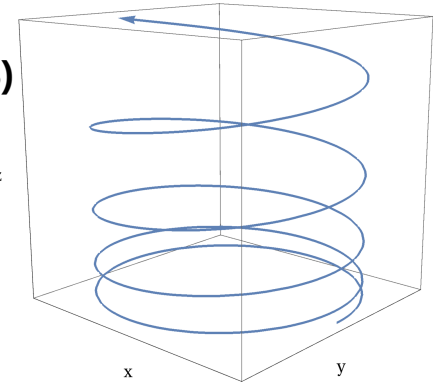
Temperature gradient

1) Pure uniformly accelerated motion

2) Rigid rotation

3)

4)



Trajectories in global equilibrium

Covariant Wigner function: a method of describing a system in quantum kinetic theory

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

Wigner function for Dirac fields

$$W(x, k)_{AB} = -\frac{1}{(2\pi)^4} \int d^4y e^{-ik \cdot y} \langle : \Psi_A(x - y/2) \bar{\Psi}_B(x + y/2) : \rangle$$



Contains thermodynamic information about the system



The interaction is weak, W has inhomogeneities only on macroscopic scales



$$W(x, k) = \frac{1}{2} \int \frac{d^3p}{\varepsilon} \left(\delta^4(k - p) U(p) f(x, p) \bar{U}(p) - \delta^4(k + p) V(p) \bar{f}^T(x, p) \bar{V}(p) \right)$$



Mean values of operators

$$\langle : \bar{\Psi}(x) A \Psi(x) : \rangle = \int d^4k \operatorname{tr}(A W(x, k))$$



$$f(x, p) = \frac{1}{e^{\beta(x) \cdot p - \xi(x)} + 1}$$

Fermi-Dirac distribution under local thermodynamic equilibrium

Ansatz of the Wigner function, taking into account the thermal vorticity tensor

For weak interacting case Wigner function is expressed in terms of: $f(x, p)$

$$\frac{1}{2m} \bar{U}(p) X(x, p) U(p) = f(x, p)$$

let's introduce X - function

Ansatz of the Wigner function

$$X(x, p) = \left(\exp[\beta_\mu p^\mu - \zeta] \exp \left[-\frac{1}{2} \varpi_{\mu\nu} \Sigma^{\mu\nu} \right] + I \right)^{-1}$$

- 1) F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, *Annals Phys.* 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].
- 2) W. Florkowski, A. Kumar and R. Ryblewski, arXiv:1806.02616 [hep-ph].
- 3) W. Florkowski, E. Speranza and F. Becattini, *Acta Phys. Polon. B49(2018)1409.*

- Used to calculate corrections in $\varpi_{\mu\nu}$ in different physical quantities.
- Used to calculate polarisation of baryons.

- Limit of zero thermal vorticity tensor

$$f(x, p) = \frac{1}{e^{\beta(x) \cdot p - \xi(x)} + 1}$$

- Nonrelativistic limit

F. Becattini and L. Tinti, Annals Phys. 325 (2010) 1566.

Generators of Lorentz transformations of spinors

$$\Sigma_{\mu\nu} = \frac{i}{4} [\gamma_\mu, \gamma_\nu]$$

- Explanation of decrease of polarisation of hyperons on STAR

Axial current and CVE

The axial current is expressed in terms of an integral with the Wigner function

$$\langle : j_\mu^5 : \rangle = -\frac{1}{16\pi^3} \epsilon_{\mu\alpha\nu\beta} \int \frac{d^3p}{\varepsilon} p^\alpha \left\{ \text{tr}(X \Sigma^{\nu\beta}) - \text{tr}(\bar{X} \Sigma^{\nu\beta}) \right\}$$

Can be calculated analytically outside perturbation theory

1) M. Buzzegoli, E. Grossi and F. Becattini, *JHEP* 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

2) F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, *Annals Phys.* 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

3) G. Prokhorov and O. Teryaev, *Phys. Rev. D* 97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].

The existence of CVE was shown

$$j_5^\mu(x) = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \omega^\mu$$

Trace calculation, technical details

- Expand in Taylor's series

$$X = \sum_{n=0}^{\infty} (-1)^n \exp \left[t(n+l) \left(\beta \cdot p - \xi - \frac{1}{2} \varpi : \Sigma \right) \right] =$$

$$\sum_{n=0}^{\infty} (-1)^n \exp \left[t(n+l) (\beta \cdot p - \xi) \right] \sum_{m=0}^{\infty} \frac{1}{m!} (t(n+l) \left(-\frac{1}{2} \varpi : \Sigma \right))^m$$

- Use the properties of gamma matrices

$$(\varpi : \Sigma)^{2k} = \eta^k \frac{1 + \gamma^5}{2} + \theta^k \frac{1 - \gamma^5}{2}, \quad k = 0, 1, 2, \dots$$

- One can find a trace in each term of the series

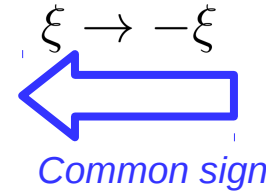
$$\begin{aligned} \text{tr} \left((\varpi : \Sigma)^{2k+1} \Sigma^{\nu\beta} \right) &= (\varpi^{\nu\beta} + i \tilde{\varpi}^{\nu\beta}) \eta^k + (\varpi^{\nu\beta} - i \tilde{\varpi}^{\nu\beta}) \theta^k \\ \text{tr} \left((\varpi : \Sigma)^{2k} \Sigma^{\nu\beta} \right) &= 0, \quad k = 0, 1, 2, \dots \end{aligned}$$

Antiparticles

$$\text{tr}(\bar{X} \Sigma^{\nu\beta})$$

Particles

$$\text{tr}(X \Sigma^{\nu\beta})$$



- Sum up the series back

Axial current, mass is not zero

- General exact formula for axial current was obtained

G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018).

G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to PhysRevD.

$$\langle j_\mu^5 \rangle = \frac{\omega_\mu + i \operatorname{sgn}(\omega a) a_\mu}{2(g_\omega - i g_a)} \int \frac{d^3 p}{(2\pi)^3} \left\{ n_F(E_p - \mu - g_\omega/2 + i g_a/2) - n_F(E_p - \mu + g_\omega/2 - i g_a/2) + n_F(E_p + \mu - g_\omega/2 + i g_a/2) - n_F(E_p + \mu + g_\omega/2 - i g_a/2) \right\} + c.c. ,$$

**Fermi-Dirac
distribution**

$$g_\omega = \frac{1}{\sqrt{2}} \left(\sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} + a^2 - \omega^2 \right)^{1/2}$$
$$g_a = \frac{1}{\sqrt{2}} \left(\sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} - a^2 + \omega^2 \right)^{1/2}$$

Axial current, mass is not zero

- In particular case $\Omega \parallel \mathbf{a}$ in comoving frame it gives

$$\langle \mathbf{j}^5 \rangle = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \left\{ n_F\left(E_p - \mu - \frac{\Omega}{2} + i\frac{|\mathbf{a}|}{2}\right) - n_F\left(E_p - \mu + \frac{\Omega}{2} + i\frac{|\mathbf{a}|}{2}\right) + n_F\left(E_p + \mu - \frac{\Omega}{2} + i\frac{|\mathbf{a}|}{2}\right) - n_F\left(E_p + \mu + \frac{\Omega}{2} + i\frac{|\mathbf{a}|}{2}\right) + c.c. \right\} \mathbf{e}_\Omega,$$

Fermi-Dirac distribution

unit vector in the direction of the angular velocity

Angular velocity and acceleration appear in combination with chemical potential

$$\mu \pm (\Omega \pm i|\mathbf{a}|)/2$$

Axial current, mass is not zero

Combination appears

$$\mu \pm (\Omega \pm i|\mathbf{a}|)/2$$

- Angular velocity $\frac{\Omega}{2}$ as a real chemical potential.
- Acceleration $i\frac{|\mathbf{a}|}{2}$ as an **imaginary** chemical potential.

Angular velocity as a chemical potential

- For $T = 0$, in region $|\mu| < m$, in stationary systems physical quantities equal zero



Let's consider

$$\alpha_\mu = 0$$



Modified chemical potential

$$\mu \pm \Omega/2$$



- Similar region with zero values of physical quantities for angular velocity. Integrals can be found analytically.

$$\langle j^5 \rangle = \frac{1}{6\pi^2} \left\{ \theta\left(\mu + \frac{\Omega}{2} - m\right) \left[\left(\mu + \frac{\Omega}{2}\right)^2 - m^2 \right]^{3/2} - \theta\left(\mu - \frac{\Omega}{2} - m\right) \left[\left(\mu - \frac{\Omega}{2}\right)^2 - m^2 \right]^{3/2} + \theta\left(-\mu + \frac{\Omega}{2} - m\right) \left[\left(\mu - \frac{\Omega}{2}\right)^2 - m^2 \right]^{3/2} - \theta\left(-\mu - \frac{\Omega}{2} - m\right) \left[\left(\mu + \frac{\Omega}{2}\right)^2 - m^2 \right]^{3/2} \right\} e_\Omega,$$



Heaviside theta-function

Angular velocity as a chemical potential

- For $T = 0$, in region $|\mu| < m$, in stationary systems physical quantities equal zero



Let's consider

$$\alpha_\mu = 0$$



Modified chemical potential

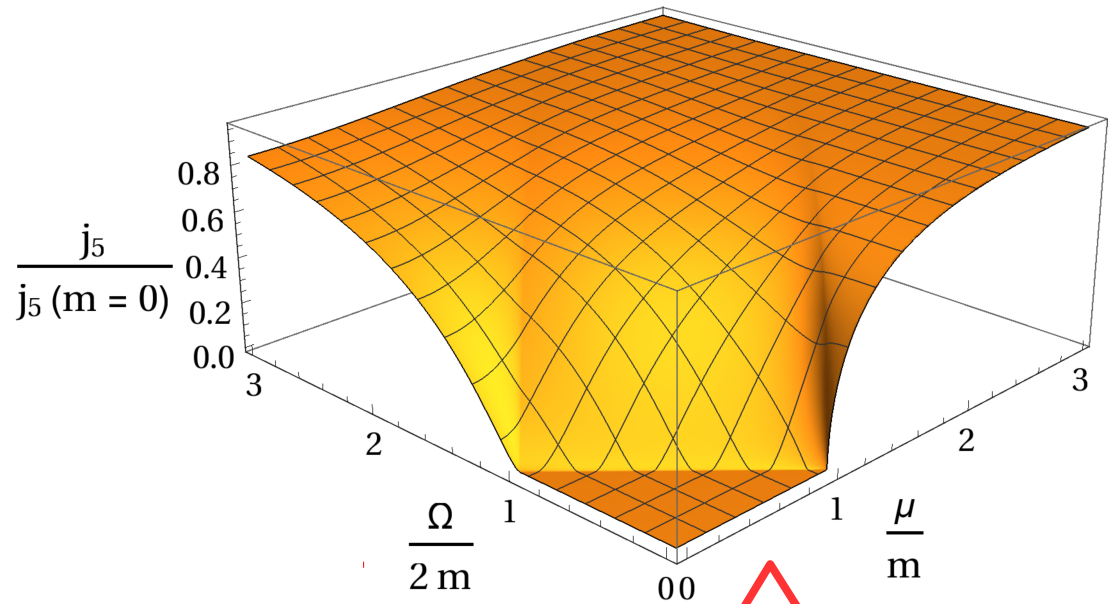
$$\mu \pm \Omega/2$$



- Similar region with zero values of physical quantities for angular velocity. Integrals can be found analytically.
- Angular velocity as a chemical potential:*

1) A. Vilenkin, *Phys. Rev. D* 20, 1807 (1979)

2) W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, *Phys. Rev. C* 97, no. 4, 041901 (2018).



Axial current equals zero at
 $\Omega < 2(m - |\mu|)$

The limit of massless fermions

- Integrals with a Fermi distribution can be found analytically in the massless limit $m \rightarrow 0$

$$\langle j_\mu^5 \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega a) a_\mu + \omega_\mu \left[-\frac{4\pi T g_a}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2} \right) \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right] - 2T^2 \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^2 + \frac{8\pi T^3 g_a}{3(g_a^2 + g_\omega^2)} \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^3 \right] + a_\mu \text{sgn}(\omega a) \left[-\frac{4\pi T g_\omega}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{g_a^2}{8\pi^2} + \frac{g_\omega^2}{8\pi^2} \right) \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right] + \frac{8\pi T^3 g_\omega}{3(g_a^2 + g_\omega^2)} \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^3 \right]$$

Remains for
 $T > \tilde{T}_U$

Nonzero for
 $T < \tilde{T}_U$

The limit of massless fermions comparison with other approaches

- In the area**
 $T > \tilde{T}_U$
one has:

$$\langle j_\mu^5 \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega a) a_\mu$$

Standard CVE, a term along the vorticity of the first order

The third-order term in vorticity $\left(-\frac{\omega^2}{24\pi^2}\right)\omega_\mu$ corresponds to

The term of order 3 along the 4-acceleration

$$\langle : j_\mu^5 : \rangle = \langle : j_\mu^5 : \rangle_{\text{Tvort}} + \langle : j_\mu^5 : \rangle_{\text{vort}} + \langle : j_\mu^5 : \rangle_{\text{acc}},$$

$$\langle : j_\mu^5 : \rangle_{\text{Tvort}} = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \omega_\mu, \quad \langle : j_\mu^5 : \rangle_{\text{vort}} = \frac{a^2 - \omega^2}{24\pi^2} \omega_\mu,$$

$$\langle : j_\mu^5 : \rangle_{\text{acc}} = \frac{1}{12\pi^2} (\omega \cdot a) a_\mu.$$

A. Vilenkin, *Phys. Rev. D* 20 (1979) 1807. doi:10.1103/PhysRevD.20.1807

A. Vilenkin, *Phys. Rev. D* 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260

M. Stone and J. Kim, *Phys.Rev. D*98 (2018) no.2, 025012 DOI: 10.1103/PhysRevD.98.025012

The limit of massless fermions

For $\Omega \parallel \mathbf{a}$, $m \rightarrow 0$, $T > \tilde{T}_U$

$$\langle \mathbf{j}^5 \rangle = \left(\frac{T^2 \Omega}{6} + \frac{(\mu + \frac{\Omega}{2} + \frac{ia}{2})^3}{12\pi^2} - \frac{(\mu - \frac{\Omega}{2} - \frac{ia}{2})^3}{12\pi^2} + \frac{(\mu + \frac{\Omega}{2} - \frac{ia}{2})^3}{12\pi^2} - \frac{(\mu - \frac{\Omega}{2} + \frac{ia}{2})^3}{12\pi^2} \right) \mathbf{e}_\Omega$$

Combination appears

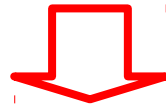
$$\mu \pm (\Omega \pm i|\mathbf{a}|)/2$$

The limit of massless fermions

Diagonalization

$$\varphi_\mu = \frac{\omega_\mu}{2\pi} + i \frac{a_\mu}{2\pi}$$

Complex superposition of acceleration and vorticity vectors



$$\langle : j_\mu^5 : \rangle = 2\pi \text{Re} \left[\left(\frac{1}{6} (T^2 - \varphi^2) + \frac{\mu^2}{2\pi^2} \right) \varphi_\mu \right]$$

- The symmetry between vorticity and acceleration is an analog of the symmetry between the magnetic and electric fields in electrodynamics

The limit of massless fermions, instabilities

$$\langle j_\mu^5 \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega a) a_\mu + \omega_\mu \left[-\frac{4\pi T g_a}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2} \right) \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right] - 2T^2 \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^2 + \frac{8\pi T^3 g_a}{3(g_a^2 + g_\omega^2)} \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^3 \right] + a_\mu \text{sgn}(\omega a) \left[-\frac{4\pi T g_\omega}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{g_a^2}{8\pi^2} + \frac{g_\omega^2}{8\pi^2} \right) \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right] + \frac{8\pi T^3 g_\omega}{3(g_a^2 + g_\omega^2)} \left[\frac{g_a}{4\pi T} + \frac{1}{2} \right]^3 \right]$$

Additional contribution for $T < \tilde{T}_U$

Depends on scalars g_ω and g_a

Temperature

$$\tilde{T}_U = \frac{g_a}{2\pi}$$

$$g_\omega = \frac{1}{\sqrt{2}} \left(\sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} + a^2 - \omega^2 \right)^{1/2}$$

$$g_a = \frac{1}{\sqrt{2}} \left(\sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} - a^2 + \omega^2 \right)^{1/2}$$

“Effective” acceleration and angular velocity



$$g_\omega = \Omega, \quad g_a = |\mathbf{a}|$$

$\Omega \parallel \mathbf{a}$

Also for zero acceleration or angular velocity

The limit of massless fermions, instabilities

$$\langle j_\mu^5 \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega a) a_\mu + \omega_\mu \left[-\frac{4\pi T g_a}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2} \right) \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor - 2T^2 \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor^2 + \frac{8\pi T^3 g_a}{3(g_a^2 + g_\omega^2)} \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor^3 \right] + a_\mu \text{sgn}(\omega a) \left[-\frac{4\pi T g_\omega}{g_a^2 + g_\omega^2} \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{g_a^2}{8\pi^2} + \frac{g_\omega^2}{8\pi^2} \right) \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor + \frac{8\pi T^3 g_\omega}{3(g_a^2 + g_\omega^2)} \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor^3 \right]$$

Instabilities due to the terms with integer part below

$$\tilde{T}_U = \frac{g_a}{2\pi} \left\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \right\rfloor$$

In particular case $\Omega = 0$ or $\Omega \parallel a$

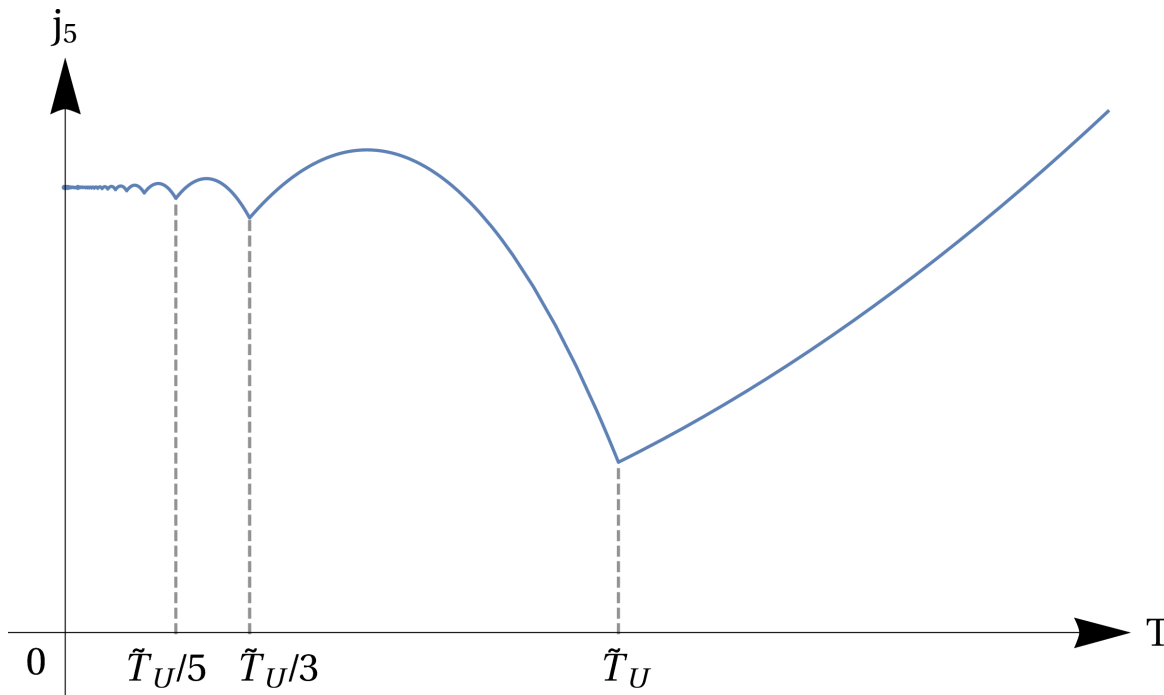
$$\tilde{T}_U \rightarrow T_U = \frac{|a|}{2\pi}$$

Unruh temperature

- Instabilities below the temperature \tilde{T}_U are a manifestation of the **Unruh-Hawking radiation**.
- From this point of view, the temperature $\tilde{T}_U(\Omega, |a|, \theta)$ should be considered as a **generalization of the Unruh-Hawking temperature** to the case of systems having simultaneously non-zero acceleration and angular velocity.

The limit of massless fermions, instabilities

- Instabilities below the temperature \tilde{T}_U are a manifestation of the **Unruh-Hawking radiation**.

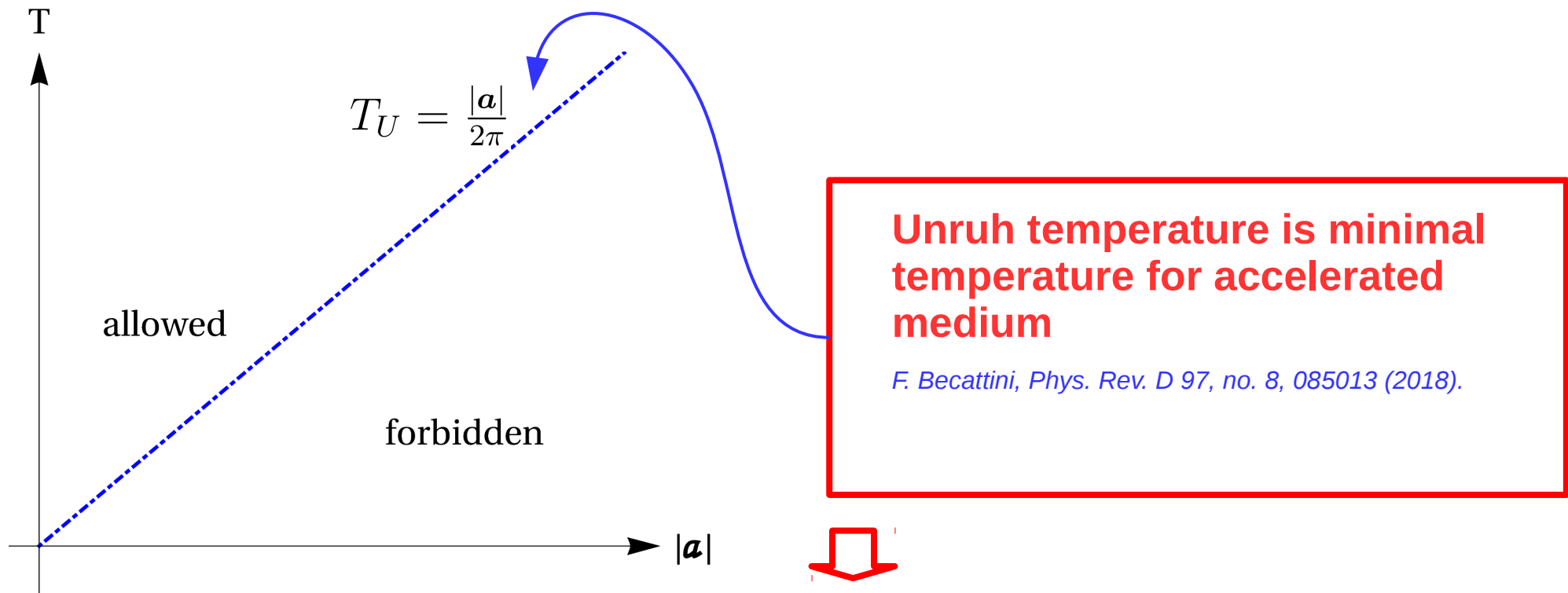


Acceleration as
imaginary chemical
potential



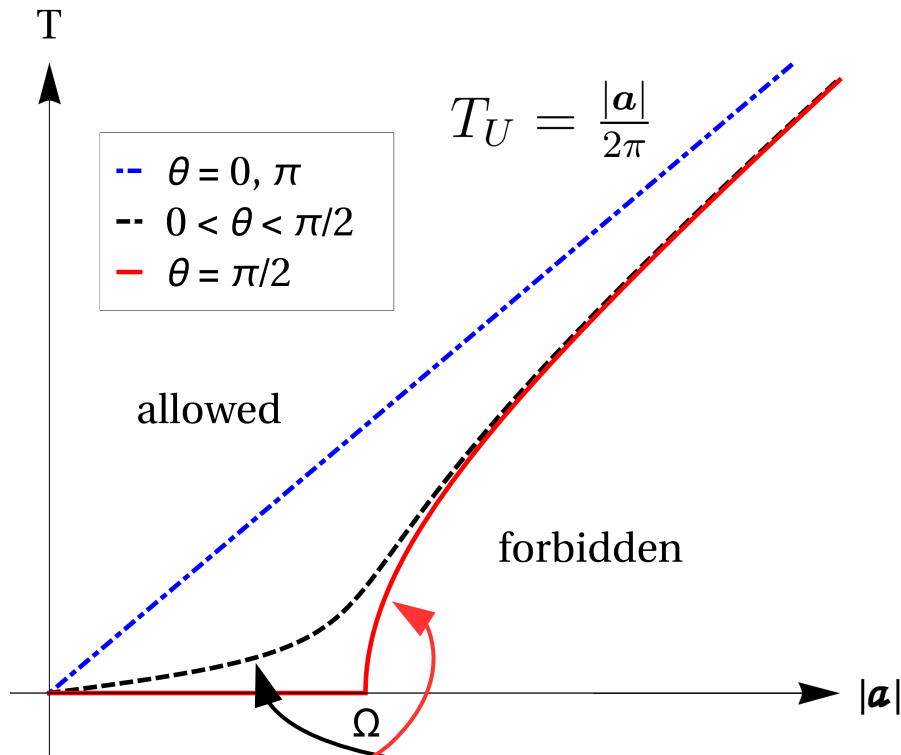
- The appearance of instabilities below Unruh temperature is a direct consequence of appearance of acceleration as an imaginary chemical potential.
- Periodic instabilities due to the terms with $\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \rfloor$ at $\frac{g_a}{2} = (2n+1)\pi T$, $n = 0, 1, 2..$ correspond to the theories with imaginary chemical potential: **Roberge-Weiss phase transitions**.
A. Roberge and N. Weiss, Nucl. Phys. B 275, 734 (1986); Y. Sakai, K. Kashiwa, H. Kouno and M. Yahiro, Phys. Rev. D 78, 036001 (2008).
- The periodicity of the axial current with an acceleration change $\frac{|a|}{2} \rightarrow \frac{|a|}{2} + 2\pi T n$ corresponds to the periodicity of the partition function in the theories with an imaginary chemical potential with respect to this potential.

Minimal temperature in the medium with acceleration and rotation



- From the analysis of quantum correlation functions for scalar fields.
- Equal zero after vacuum subtraction.
- In fact, rephrasing of Unruh effect.

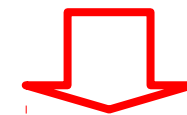
Minimal temperature in the medium with acceleration and rotation



What will be for fermions?

Look: W. Florkowski, E. Speranza and F. Becattini, arXiv:1803.11098 [nucl-th], for fermions bound temperature is twice Unruh temperature

What will change, if we consider acceleration and rotation simultaneously?



Let's use the **absence of instabilities** in axial current as a criterion to define minimal possible temperature

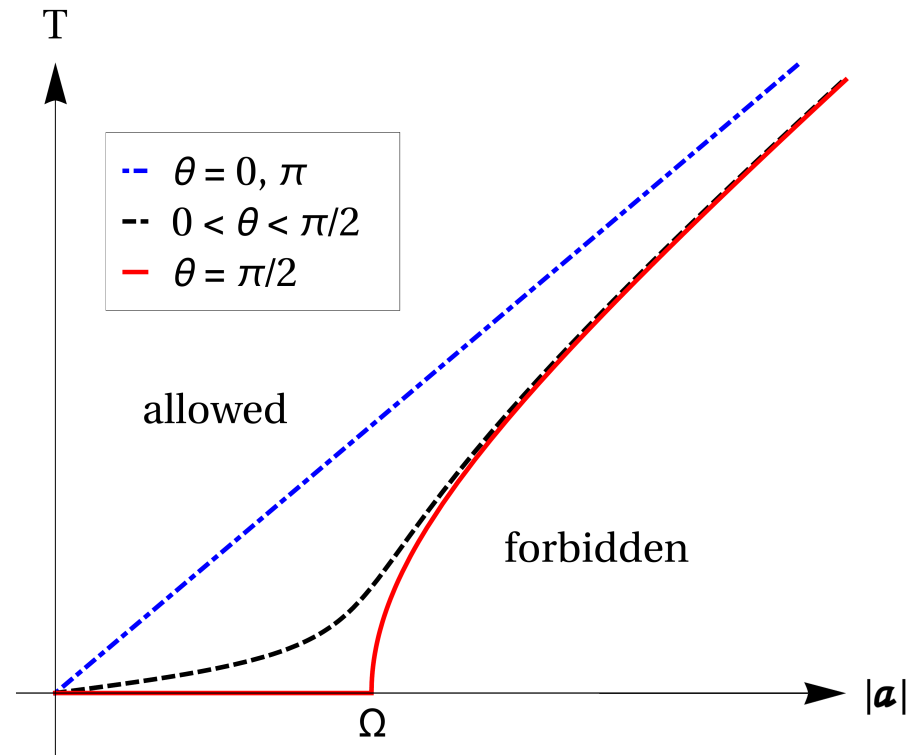
For $\Omega \perp a$:

$$\tilde{T}_U = \frac{\sqrt{|a|^2 - \Omega^2}}{2\pi}$$

Minimal temperature for systems with acceleration and rotation:

$$\tilde{T}_U = \frac{g_a}{2\pi}$$

Minimal temperature in the medium with acceleration and rotation

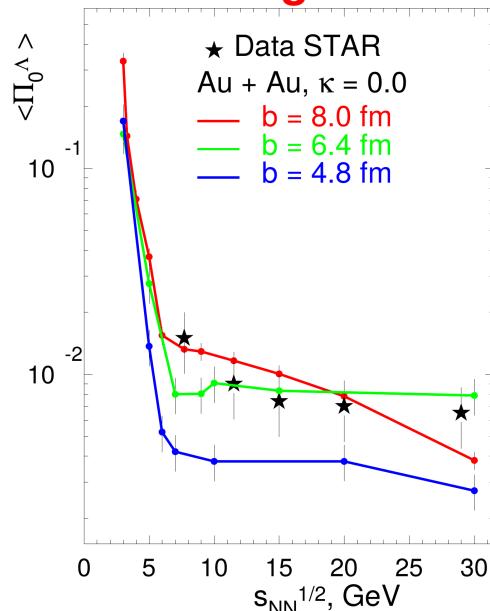


- Minimal temperature shifts down for the medium with rotation and acceleration.
- No bound temperature for $\Omega \perp a$ if $|a| < \Omega$.

2 approaches to the calculation of the polarization of hyperons

1-st approach

The fall of polarization of hyperons with increasing collision energy.



- Sorin, Alexander et al. *Phys.Rev. C*95 (2017) no.1, 011902 arXiv:1606.08398 [nucl-th]
- M. Baznat, K. Gudima, A. Sorin and O. Teryaev, *EPJ Web Conf.* 138 (2017) 01008.doi:10.1051/epjconf/201713801008; arXiv:1701.00923

The polarization is determined by the axial charge of strange quarks

from CVE

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6},$$

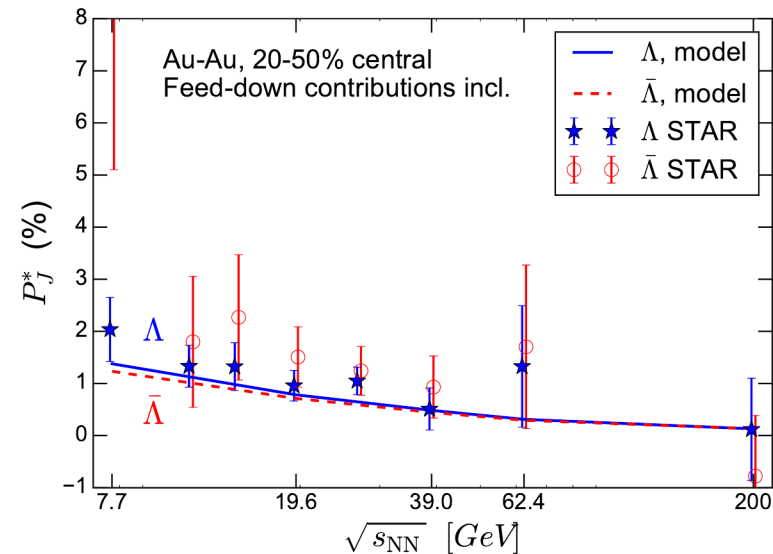
$$Q_5^s = N_c \int d^3x C(r) c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

2-nd approach

- Karpenko, Iu. et al. *Nucl.Phys. A*967 (2017) 764-767 arXiv:1704.02142 [nucl-th]

Based on the Wigner function

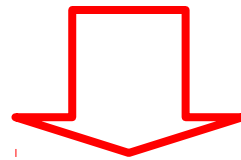
$$S^\mu(x, p) = -\frac{1}{8m} (1 - f(x, p)) \epsilon^{\mu\nu\rho\sigma} p_\sigma \varpi_{\nu\rho}$$



The fall of polarization is caused by the decrease of the strange chemical potential

2 approaches to the calculation of the polarization of hyperons

- It was proved that the Wigner function used in the second approach leads to CVE.
- CVE underlies the first approach.
- Thus, CVE is essential for both approaches, which can explain the same polarization behaviour in them.



There is a connection between the two approaches to the calculation of the polarization of hyperons.

Conclusions

- **It is shown that CVE follows from the covariant Wigner function for the medium with thermal vorticity** (the same result was given previously in *M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091.*).
- A formula for the axial current in a rotating and accelerated medium is derived outside the limits of perturbation theory from the Wigner function. It includes higher order corrections to CVE.
- **It is shown that the angular velocity plays the role of a chemical potential, while the acceleration appears as an imaginary chemical potential** (in particular case when $\Omega \parallel \mathbf{a}$, in general case scalars g_ω and g_a appear).
- At zero temperature, the axial current, as a function of the angular velocity and chemical potential, vanishes in a two-dimensional plane region $\Omega < 2(m - |\mu|)$.
- Instabilities in axial current below $\tilde{T}_U = \frac{g_a}{2\pi}$, indicating the Unruh effect.
- The assumption that the "effective" Unruh temperature $\tilde{T}_U = \frac{g_a}{2\pi}$, depending on acceleration and angular velocity, is the lower boundary temperature (based on the requirement of no instabilities).

Thank you for attention!

The second approach: the equilibrium density operator

- Thermodynamics can be derived from field theory, constructed in terms of the path integral in imaginary time
- The central role is played by the density operator
- **General covariant form of the density operator for a medium in local thermodynamic equilibrium**

$$\hat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma} d\Sigma_{\mu} \left(\hat{T}^{\mu\nu}(x) \beta_{\nu}(x) - \zeta(x) \hat{j}^{\mu}(x) \right) \right] \Rightarrow \text{Maximum } S = -\text{tr}(\hat{\rho} \log \hat{\rho})$$

D.N. Zubarev, A.V. Prozorkevich and S.A. Smolyanskii, Derivation of nonlinear generalized equations of quantum relativistic hydrodynamics, Theor. Math. Phys. 40 (1979) 821.

Provided that $\nabla_{\mu} \beta_{\nu} + \nabla_{\nu} \beta_{\mu} = 0 \quad \nabla_{\mu} \zeta = 0 \Rightarrow$

$$\begin{aligned} \beta_{\mu}(x) &= b_{\mu} + \varpi_{\mu\nu} x^{\nu} & \zeta &= \text{const.} \\ b_{\mu} &= \text{const} & \varpi_{\mu\nu} &= \text{const} \end{aligned}$$

$\hat{\rho}$ does not depend on the choice of a hypersurface $d\Sigma_{\mu}$

global thermodynamic equilibrium

The density operator for a medium with a thermal vorticity tensor

Global equilibrium conditions

$$\nabla_{\mu}\beta_{\nu} + \nabla_{\nu}\beta_{\mu} = 0 \quad \nabla_{\mu}\zeta = 0$$

The form of the density operator for a medium with rotation and acceleration

$$\hat{\rho} = \frac{1}{Z} \exp \left[-b_{\mu} \hat{P}^{\mu} + \frac{1}{2} \varpi_{\mu\nu} \hat{J}^{\mu\nu} + \zeta \hat{Q} \right]$$

Thermal vorticity tensor

- *F. Becattini and E. Grossi, Phys. Rev. D 92 (2015) 045037 [arXiv:1505.07760] [INSPIRE].*
- *F. Becattini, arXiv:1712.08031 [gr-qc], to appear in Phys. Rev. D.*

4-momentum operator

Generators of Lorentz transformations

$$\hat{J}^{\mu\nu} = \int_{\Sigma} d\Sigma_{\lambda} \left(x^{\mu} \hat{T}^{\lambda\nu} - x^{\nu} \hat{T}^{\lambda\mu} \right)$$

Charge operator

Пределные случаи

$$\hat{\rho} = \frac{1}{Z} \exp \left[-b_\mu \hat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \hat{J}^{\mu\nu} + \zeta \hat{Q} \right]$$

$$\varpi = 0$$

Пределные случаи

$$\alpha_\mu = 0$$

$$\hat{\rho} = \frac{1}{Z} \exp[-b \cdot \hat{P} + \zeta \hat{Q}]$$

Великое каноническое распределение

$$\hat{\rho} = \frac{1}{Z} \exp \left[-\hat{H}/T_0 + \omega_0 \hat{J}_z/T_0 \right]$$

V.E. Ambrus and E. Winstanley, Rotating fermions inside a cylindrical boundary, Phys. Rev. D 93 (2016) 104014 [arXiv:1512.05239] [INSPIRE].

Mean value of the physical quantity operator

Mean in terms of the path integral

$$\langle \hat{O}(x) \rangle = \text{tr}(\hat{\rho} \hat{O}(x))_{\text{ren}}$$

Statistical sum: reduction of disconnected correlators

Perturbation theory in the third order

$$\begin{aligned} \langle \hat{O}(x) \rangle &= \langle \hat{O}(x) \rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|} \int_0^{|\beta|} d\tau \langle T_\tau J_{-i\tau u}^{\mu\nu} \hat{O}(0) \rangle_{\beta(x),c} \\ &+ \frac{\varpi_{\mu\nu} \varpi_{\rho\sigma}}{8|\beta|^2} \int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J_{-i\tau_x u}^{\mu\nu} J_{-i\tau_y u}^{\rho\sigma} \hat{O}(0) \rangle_{\beta(x),c} \\ &+ \frac{\varpi_{\mu\nu} \varpi_{\rho\sigma} \varpi_{\alpha\beta}}{48|\beta|^3} \int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J_{-i\tau_x u}^{\mu\nu} J_{-i\tau_y u}^{\rho\sigma} J_{-i\tau_z u}^{\alpha\beta} \hat{O}(0) \rangle_{\beta(x),c} + \dots \end{aligned}$$

Ordering by imaginary time

Connected correlators

$$\langle \hat{J} \hat{O} \rangle_c = \langle \hat{J} \hat{O} \rangle - \langle \hat{J} \rangle \langle \hat{O} \rangle$$

- Reduction of the members by vorticity, starting with order 4 in the approach with the Wigner function and in

A. Vilenkin, *Phys. Rev. D* 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260

Axial current in the third order of perturbation theory

Three types of admissible parity:

$$\langle \hat{j}_5^\lambda(x) \rangle_3 = A_1 w^2 w^\lambda + A_2 \alpha^2 w^\lambda + A_3 (w\alpha) \alpha^\lambda$$

Violates the conservation of axial charge

The main points of the technique of computing of hydrodynamic coefficients

- Representation of composite operators in a split form (*point splitting*)

$$\hat{T}_{\mu\nu}(X) = \lim_{X_1, X_2 \rightarrow X} \mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) \bar{\Psi}(X_1) \Psi(X_2)$$

$$\mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) = \frac{i^{\delta_{0\mu} + \delta_{0\nu}}}{4} [\tilde{\gamma}_\mu(\partial_{X_2} - \partial_{X_1})_\nu + \tilde{\gamma}_\nu(\partial_{X_2} - \partial_{X_1})_\mu]$$

M. Buzzegoli, E. Grossi and F. Becattini, *JHEP* 1710 (2017) 091
doi:10.1007/JHEP10(2017)091
[arXiv:1704.02808 [hep-th]].

- Summation over the Matsubara frequencies

$$\frac{1}{\beta} \sum_{\{\omega_n\}} \frac{(\omega_n \pm i\mu)^k e^{i(\omega_n \pm i\mu)\tau}}{(\omega_n \pm i\mu)^2 + E^2}$$

$$= \frac{1}{2E} \left[(-iE)^k e^{\tau E} (\theta(-\tau) - n_F(E \pm \mu)) + (iE)^k e^{-E\tau} (\theta(\tau) - n_F(E \mp \mu)) \right]$$

$$= \frac{1}{2E} \sum_{s=\pm 1} (-isE)^k e^{\tau s E} [\theta(-s\tau) - n_F(E \pm s\mu)]$$

Fermi distribution

Детали вычислений

Термальный пропагатор фермионов:

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

$$\langle T_\tau \Psi_a(X) \bar{\Psi}_b(Y) \rangle = \sum_{\{P\}} e^{iP^+ \cdot (X-Y)} \frac{(-i\not{P}^+ + m)_{ab}}{(P^+)^2 + m^2} = \sum_{\{P\}} e^{iP^+ \cdot (X-Y)} (-i\not{P}^+ + m)_{ab} \Delta(P^+)$$

$$\Delta(P^\pm) = \frac{1}{P^{\pm 2} + m^2}$$

Интегрирование по фазовому пространству и суммирование по мацубаровским частотам

Явная зависимость от координат переписывается в виде производных по импульсу:

$$\int d^3x d^3y d^3z f(\mathbf{p}, \mathbf{q}, \mathbf{k}, \mathbf{r}) e^{-i\mathbf{p}(\mathbf{x}-\mathbf{y}) - i\mathbf{q}(\mathbf{x}-\mathbf{z}) - i\mathbf{k}\mathbf{y} - i\mathbf{r}\mathbf{z}} x^i y^j z^k =$$

$$i \left(\frac{\partial^3}{\partial r^k \partial k^j \partial p^i} + \frac{\partial^3}{\partial r^k \partial k^j \partial k^i} \right) f(\mathbf{p}, \mathbf{q}, \mathbf{k}, \mathbf{r}) \Big|_{\substack{\mathbf{q} = -\mathbf{p} \\ \mathbf{k} = \mathbf{p} \\ \mathbf{r} = -\mathbf{p}}}$$

Axial current in the third order of perturbation theory

Axial current in the third order of perturbation theory

$$\langle \hat{j}_5^\lambda(x) \rangle_3 = A_1 w^2 w^\lambda + A_2 \alpha^2 w^\lambda + \cancel{A_3 (w\alpha) \alpha^\lambda}$$

The results of calculating the coefficients

$$A_1 = -\frac{1}{24|\beta|^3\pi^2}$$

$$A_2 = -\frac{1}{8|\beta|^3\pi^2}$$

$$A_3 = 0$$



The axial charge is conserved

Comparison with the result obtained using the Wigner function

Calculation based on the density operator in the framework of a QFT at a finite temperature

$$j_5^\lambda(x) = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2} \right) \omega^\lambda$$

Calculation based on the Wigner function

$$\langle : j_\mu^5 : \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega \cdot a) a_\mu$$

- Cubic terms coincide.
- Coincide linear terms corresponding to CVE.
- The second current has a term along the acceleration vector, which violates the conservation of the axial charge.
- The coefficients that stand before $a^2 \omega^\lambda$ are different.

Запланированные задачи

- Сравнение двух подходов к расчёту поляризации Λ -гиперонов: вывод связи поляризации с аксиальным током.
- Исследование вопроса о неперенормируемости киральных эффектов в теории поля при конечных температурах.
- Исследование влияния эффектов, связанных с гравитацией и, в частности, с гравитационной аномалией, на физику киральных жидкостей.
- Дополнительная проверка сохранения аксиального заряда.

$$\begin{aligned} \text{tr}(X\Sigma^{\nu\beta}) = & \left\{ (\exp [(\beta \cdot p - \xi - g_1 + ig_2)] + 1)^{-1} - (\exp [(\beta \cdot p - \xi + g_1 - ig_2)] + 1)^{-1} \right\} \\ & \frac{1}{4(g_1 - ig_2)} [\varpi^{\nu\beta} - i \text{sgn}(\varpi : \tilde{\varpi}) \tilde{\varpi}^{\nu\beta}] + \\ & \left\{ (\exp [(\beta \cdot p - \xi - g_1 - ig_2)] + 1)^{-1} - (\exp [(\beta \cdot p - \xi + g_1 + ig_2)] + 1)^{-1} \right\} \end{aligned}$$

$$\begin{aligned} C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = & \frac{1}{|\beta|^3} \int d\tau_x d\tau_y d\tau_z d^3x d^3y d^3z \langle T_\tau \hat{T}^{\alpha_1\alpha_2}(\tau_x, \mathbf{x}) \\ & \hat{T}^{\alpha_3\alpha_4}(\tau_y, \mathbf{y}) \hat{T}^{\alpha_5\alpha_6}(\tau_z, \mathbf{z}) \hat{j}_5^\lambda(0) \rangle_{\beta(x),c} x^i y^j z^k \end{aligned}$$

$$\begin{aligned} A_1 = & -\frac{1}{6} (C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + \\ & C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222}) \end{aligned}$$

$$\langle T_\tau \hat{T}^{\alpha_1 \alpha_2}(\tau_x, \mathbf{x}) \hat{T}^{\alpha_3 \alpha_4}(\tau_y, \mathbf{y}) \hat{T}^{\alpha_5 \alpha_6}(\tau_z, \mathbf{z}) \hat{j}_5^\lambda(0) \rangle_{\beta(x), c} = \lim_{\substack{X_1, X_2 \rightarrow X \\ Y_1, Y_2 \rightarrow Y \\ Z_1, Z_2 \rightarrow Z \\ F_1, F_2 \rightarrow F=0}} \mathcal{D}_{a_1 a_2}^{\alpha_1 \alpha_2}(\partial_{X_1}, \partial_{X_2})$$

$$\mathcal{D}_{a_3 a_4}^{\alpha_3 \alpha_4}(\partial_{Y_1}, \partial_{Y_2}) \mathcal{D}_{a_5 a_6}^{\alpha_5 \alpha_6}(\partial_{Z_1}, \partial_{Z_2}) \mathcal{J}_A^\lambda \langle \bar{\Psi}_{a_1}(X_1) \Psi_{a_2}(X_2) \bar{\Psi}_{a_3}(Y_1) \Psi_{a_4}(Y_2) \bar{\Psi}_{a_5}(Z_1) \Psi_{a_6}(Z_2) \bar{\Psi}_{a_7}(F_1) \Psi_{a_8}(F_2) \rangle_{\beta(x), c}$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$



Разложение на генераторы углового момента и сдвига

$$\hat{J}^{\mu\nu} = u^\mu \hat{K}^\nu - u^\nu \hat{K}^\mu - u_\rho \epsilon^{\rho\mu\nu\sigma} \hat{J}_\sigma$$

Аксиальный ток в третьем порядке теории возмущений

Каждый из гидродинамических коэффициентов представляется как сумма величин C

$$A_1 = -\frac{1}{6} (C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222})$$

Каждый из этих коэффициентов имеет вид:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{i}{|\beta|^3(2\pi)^3} \int [d\tau] d^3p \left(\frac{\partial^3 D_1}{\partial r^k \partial k^j \partial p^i} + \frac{\partial^3 D_1}{\partial r^k \partial k^j \partial k^i} \right) \Big|_{\substack{\mathbf{q}=-\mathbf{p} \\ \mathbf{k}=\mathbf{p} \\ \mathbf{r}=-\mathbf{p}}} + \dots$$

$$D_1 = \frac{-1}{16E_p E_q E_k E_r} \sum_{\substack{s_1, s_2, s_3, \\ s_4 = \pm 1}} e^{(\tau_x - \tau_y)s_1 E_p + (\tau_x - \tau_z)s_2 E_q + \tau_y s_3 E_k + \tau_z s_4 E_r} A^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\lambda}(\tilde{P}, \tilde{K}, \tilde{Q}, \tilde{R})$$

$$\left(\Theta(-s_1[\tau_x - \tau_y]) - n_F(E_p - s_1\mu) \right) \left(\Theta(-s_2[\tau_x - \tau_z]) - n_F(E_q + s_2\mu) \right) \\ \left(\Theta(-s_3) - n_F(E_k - s_3\mu) \right) \left(\Theta(-s_4) - n_F(E_r + s_4\mu) \right)$$

Многочлен от аргументов

$$C^{02|02|02|3|111} = -C^{01|01|01|3|222} = \frac{29}{80|\beta|^3\pi^2}$$

$$C^{01|02|02|3|211} = C^{02|01|02|3|121} = C^{02|02|01|3|112} = -C^{02|01|01|3|122} =$$

$$-C^{01|02|01|3|212} = -C^{01|01|02|3|221} = \frac{19}{240|\beta|^3\pi^2}$$

The covariant Wigner function conclusion

- The result obtained is in full accordance with all known theoretical calculations of the CVE (the same conclusion in M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].).
- The simplicity of the formula obtained.
- The cubic term for the vorticity exactly coincides with the other two conclusions for the axial current made by A. Vilenkin.
- Symmetry between vorticity and acceleration.
- Non-conservation of the axial charge in the case when the acceleration and the rotation speed are not equal to zero and not perpendicular.
- An ansatz for the Wigner function is used.
- An approximation is used for the Wigner function.



AN ADDITIONAL INDEPENDENT APPROACH FOR THE INSPECTION OF THE OBTAINED RESULT SHOULD BE USED

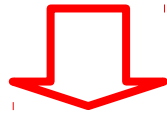
Учёт аксиального химического потенциала в первом порядке теории возмущений

Термальный пропагатор с учётом векторного и аксиального химического потенциалов: случай массы равной нулю

$$\langle T_\tau \Psi_a(X) \bar{\Psi}_b(Y) \rangle_{\beta(x)} = \int_{\{P\}} \left[e^{iP^{+-}(X-Y)} \mathcal{P}_+ \frac{\tilde{\gamma}_\alpha(-iP_\alpha^{+-})}{(P^{+-})^2} + e^{iP^{++}(X-Y)} \mathcal{P}_- \frac{\tilde{\gamma}_\alpha(-iP_\alpha^{++})}{(P^{++})^2} \right]$$

$$\mu_5 \neq 0$$

$$m = 0$$



$$\mathcal{P}_\pm = \frac{1 \pm \gamma_5}{2}$$

$$P^{+-\lambda} = (p_n + i\mu - i\mu_5, \mathbf{p})$$

Аксиальный ток в первом порядке

$$j_5^\lambda(x) = n_5 u^\lambda + \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2} \right) \omega^\lambda$$

Векторный ток в первом порядке

$$j^\lambda = n u^\lambda + \frac{\mu\mu_5}{\pi^2} \omega^\lambda$$



Соответствует другим подходам: эффективная теория поля с аксиальными аномалиями и кинетический

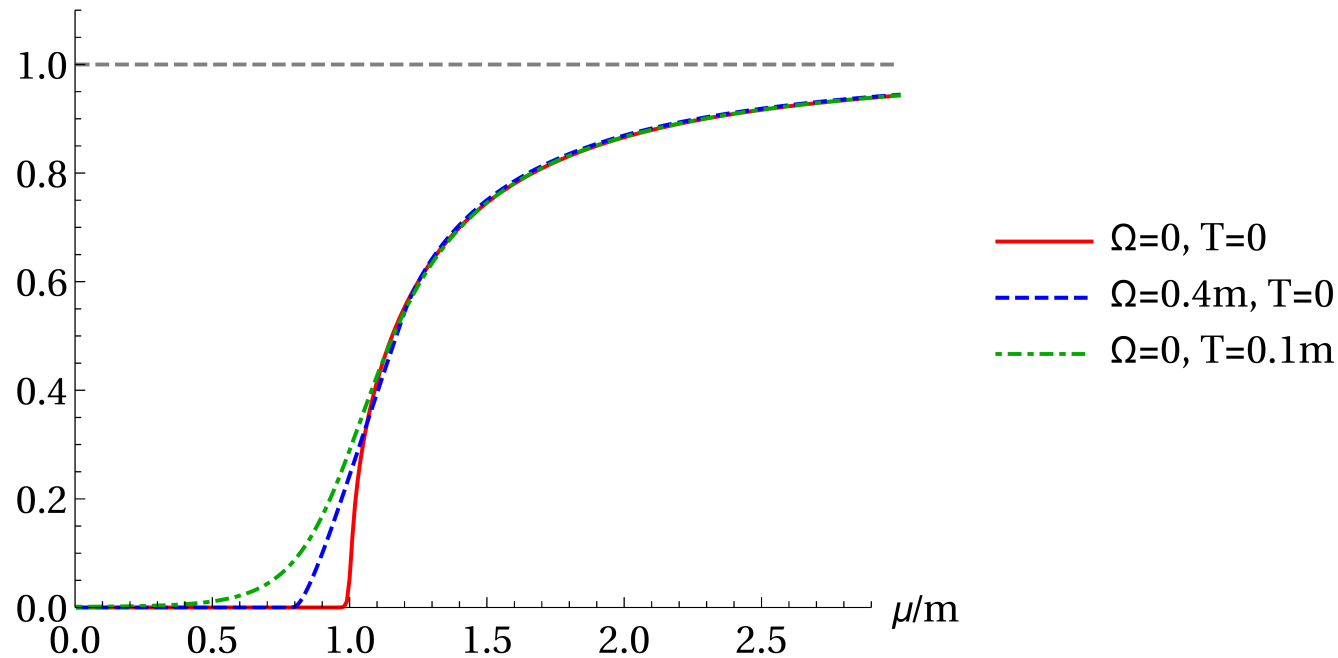


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- J. H. Gao, Z. T. Liang, S. Pu, Q. Wang and X. N. Wang, *Phys. Rev. Lett.* 109 (2012) 232301 doi:10.1103/PhysRevLett.109.232301 [arXiv:1203.0725 [hep-ph]].

Axial current, mass is not zero

$$\frac{W_A(\mu, T, m, \Omega)}{T^2/6 + \mu^2/(2\pi^2)}$$

$$\alpha_\mu = 0$$



Axial current, as a function of the chemical potential at different values of the rotational speed and temperature

- *The presence of a step at a chemical potential equal to the mass*
- *The step is smoothed with increasing temperature or speed of rotation*